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The Challenges of Surface Reconstruction

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1 Introduction

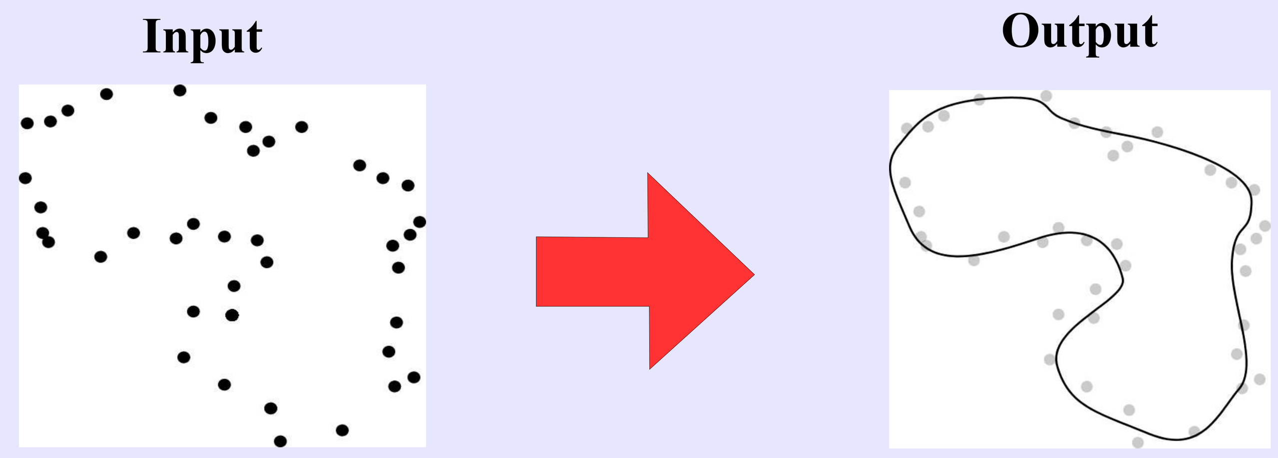
1.1 B-rep model

There exist two main families of models for 3D objects:

- CSG tree, it is tree that leaves are primitive shapes (polytopes, cones, spheres...) and that nodes are operators (symmetric difference, union, intersection...)
- B-rep model, it is a description of the boundary of the object.

The boundary is made of several surfaces (B-splines, Béziers...) having borders made of edges and vertices. The B-rep model contains information of topological nature to describe how the edges and vertices of each surface are connected to the others.

1.2 Input and Output



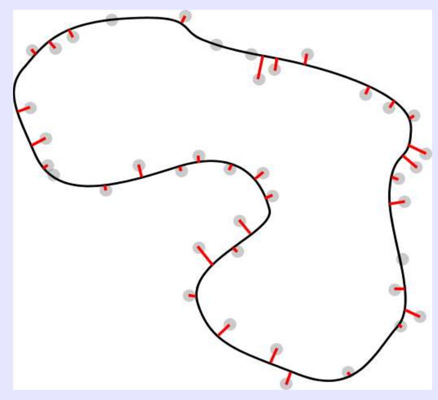
An unorganized points cloud $S \subset \mathbb{R}^d$

A surface made of parametric surfaces used in CAD (Beziers, B-spline, NURBS...)

Given an object modelized by a B-rep model, we can compute points belonging to its boundary. The problem of *surface reconstruction* is the inverse problem consisting in constructing a B-rep model from the points set. It is a practical problem of reverse engineering which arises each time that a model of a 3D object has to be computed from the points cloud measured by a 3D scanner...

1.3 A problem of Approximation

The problem is more to find a boundary which is close to the initial points set than a surface which matches exactly with the input

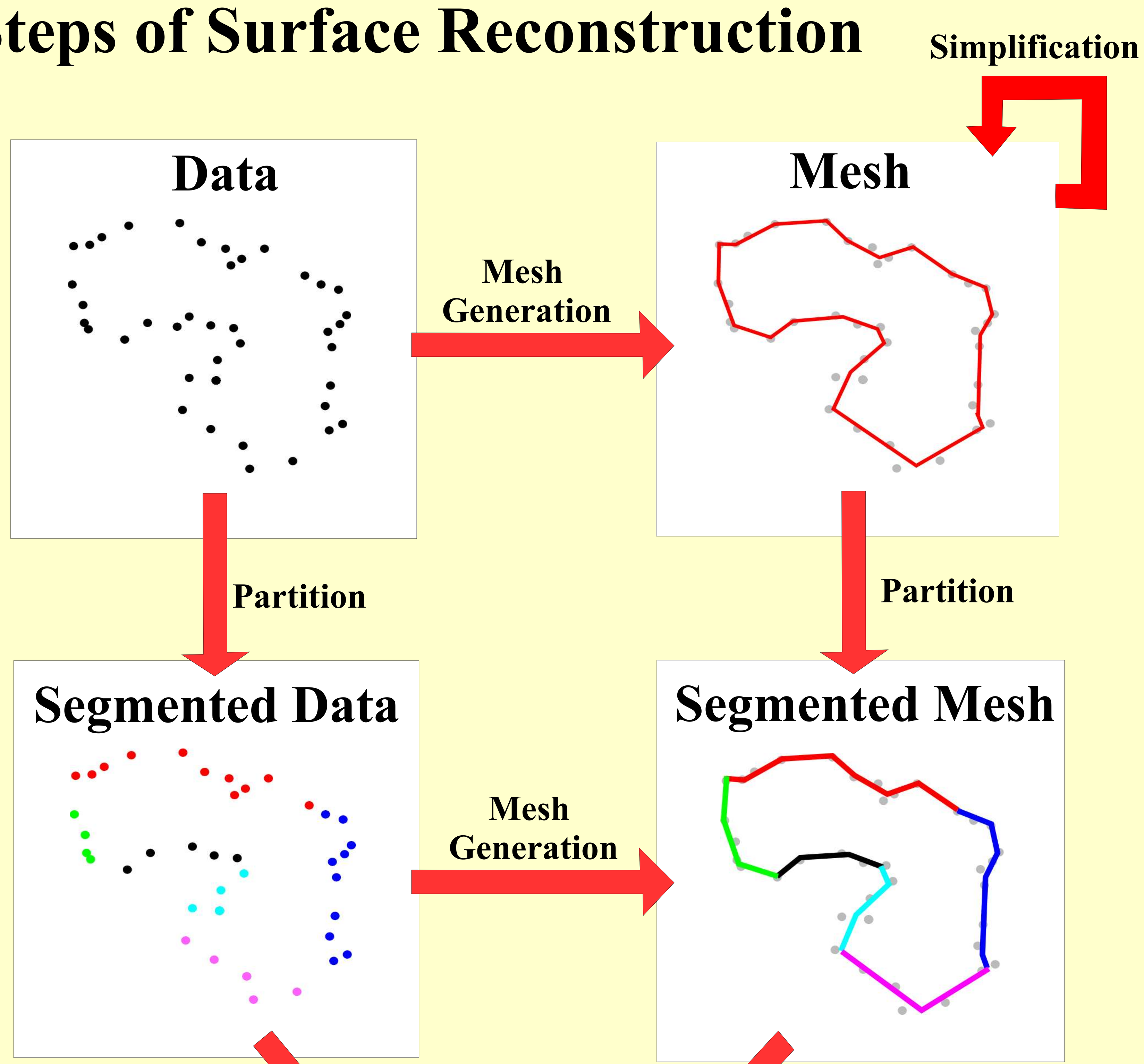


Approximation allows to decrease the degree of freedom necessary to fit exactly the input. It leads however to a problem of reconstruction which is highly unlinear and that number of variables is too high to considerate a global research of a minimal error. Mathematical expression of the problem requires to introduce a norm on the error vector which can be as well euclidian or uniform if we try to obtain a surface with a maximal accuracy on each point..

1.4 Purpose

We do not pretend to be original on the question.We hope however to put in perspective the main steps of surface reconstruction as considered in H. Hoppe PhD Thesis *Surface Reconstruction from unorganized points* (1992) or in several ulterior works with M. Eck and others authors. Ideas on the reconstruction process can also be found in *Advanced Surface Fitting Techniques* (2002) by V. Weiss, L. Andor, G. Renner, T. Varady. We find many details on the parametrization step in excellent M.S. Floater and K. Hormann survey on the question *Surface Parametrization: a tutorial and survey* (2005).

2 The four Main Steps of Surface Reconstruction



2.1 Data

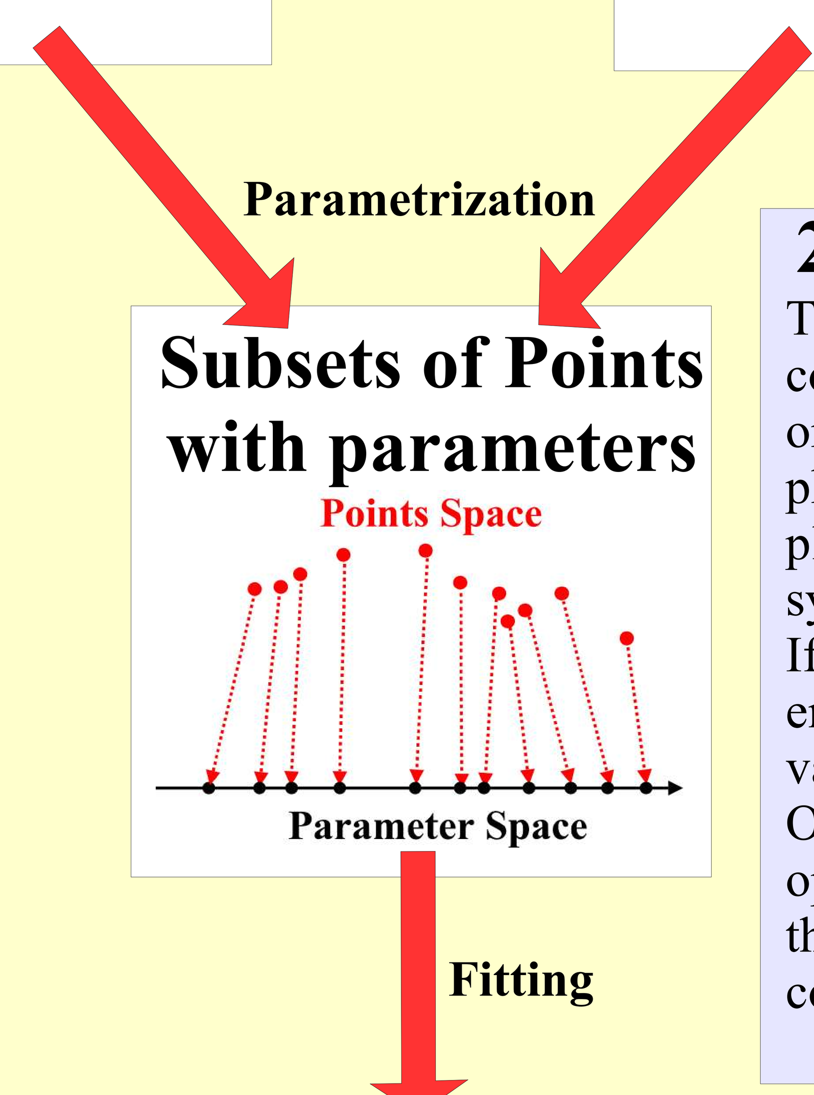
Raw data comes usually from a measurements. A preprocessing step can be needed to delete points with errors or noise that amplitude can perturb the reconstruction.
The density of the points on the surface to be reconstructed is of course determining the quality of the reconstruction. LP-fitting method can however deal (theoritically) with irregular sampling.
In case of accurate data, the input can be made of a set of voxels having properties of digital topology which allow to use specific algorithms such as Marching Cubes.

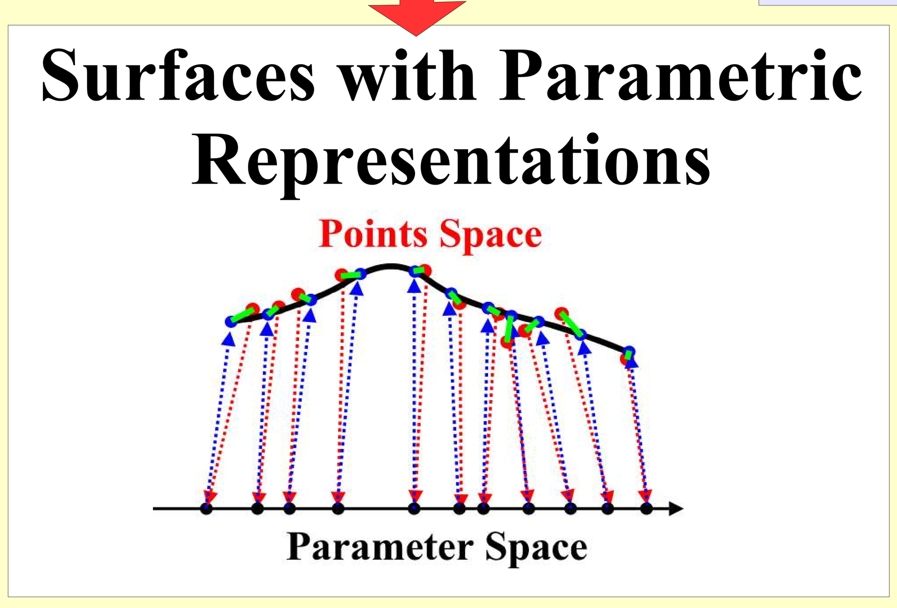
2.2 Mesh Generation

The first point to discuss is the possibility that points of the input could be lost in the sense that they are not vertices of the mesh. Should we accept it?
The answer to this question is critical since it determines the class of mesh generator that should be used.
In case of a points set made of voxels, Marching Cubes is a classical method. It provides small triangles. It is an advantage if we take care of keeping all the points of the input but it can be a drawback otherwise (it can however be completed by a Simplification step to obtain larger triangles).
Still in the case of points set made of voxels, algorithms coming from the field of Digital Geometry are waited for the coming months...
In the case of an arbitrary points cloud, we can think of using the notion of computationnal geometry called alpha-shape. If some points are coplanar, this method has the advantage to provide a triangulation leading to Delaunay. It leads however to the problem to find a good value for alpha. With this tool, we have practical problems to generate a mesh homeomorph to a surface.
There exist of course many other approaches that should be investigated in the framework of surface reconstruction but by lack of time, our knowledge on the topic remains perhaps a bit archaic...

2.3 Partition

The partition step is the one which generates the topological information on how the surfaces, edges and vertices should be related to each other. This knowledge of the reconstructed surface is directly derived from the adjacency relations between the regions of the partition. If a region has 4 adjacent regions, the surface reconstructed from this region should have 4 boundaries with the neighboring surfaces.
The usual patches of surfaces used in CAD (Beziers, B-splines, NURBS) have 4 borders in the classical case. We can also consider triangular Béziers surfaces (with 3 borders) but it would be unusual to work with surfaces having more borders. Thus it seems to be complicated to join a surface fitting the points of a region with more than 4 adjacent surfaces corresponding to the neighboring regions. It is probably easier to construct regions which have no more than 4 adjacent regions.
One of the possible approach in the framework of mesh partition, is to consider the problem of segmentation as an application of mesh simplification.
The question to obtain a simplified mesh satisfying the constraint on the number of adjacency regions remains however open (as far as we know):
Does there exist methods of segmentation with constraints on the adjacency graph of the regions ?





2.4 Parametrization

The problem is to associate to each points the parameters which should correspond to it in the reconstructed surface. It means embeding the set of points or the mesh belonging to each region in a plane. The usual principle is quite physical. We relate the points by springs before embedding the network in the plane by fixing the boundary points on a convex border (it leads to a linear system).
If we add that the construction should be shape preserving (coplanar points are embedded in themselves), the straightness of the springs should have specific values.
One of the possible choices for a mesh is called harmonic but if the sum of opposite angles is greater than Pi, then the straightness becomes negative with the drawback that the embedded mesh can overlap. Some methods can be considered to avoid this bad configuration (flip the triangles, introduce points) but it leads to the general problem of generating meshes satisfying the condition that the sum of opposite angles is less than Pi, knowing that in a plane this condition characterizes exactly the Delaunay triangulation...

2.5.1 LSQ and LP Fitting

Last step. Compute the surfaces corresponding to each region. The problem is to minimize the norm of the error vector (in green) that coordinates are the differences of coordinates between the initial points and the points of the surface having the parameters computed in previous step. The usual approach is to work with euclidian norm (LSQ-Fitting) but it can be better in case of irregular sampling to choose uniform approach which leads to solve a Linear Program (LP-Fitting).

2.5.2 Joining the surfaces

Linear constraints should be added in the fitting instance to guarantee the joining of the reconstructed surfaces.