LP fitting approach for reconstructing parametric surfaces from points clouds

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Introduction

Geometric modeling

Many ways to represent an object in a computer

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An object can be modelized as

- a point cloud
- a voxels set
- a mesh
- parametric surfaces (Bézier, B-Spline)

$\downarrow$ complex level

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A problem of geometric modeling

A problem of reverse engineering is passing from point cloud (issued from a scanner) to parametric surface (CAD)

Introduction

[1 Eck & Hoppe]

3D point cloud

↓ mesh generation

mesh

↓ partition

set of meshes (homeomorphic to discs)

↓ parameterization

parameterized point cloud

↓ least squares fitting

parametric surface

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LP fitting ↓ least squares fitting

parametric surface

Outline

Definition of some parametric surfaces

A definition of some parametric surfaces

$$Q(s,t) = \sum_{i=1}^{n} P_i f_i(s,t)$$

with

- Basis of fonctions $f_i : \mathbb{R}^2 \mapsto \mathbb{R}$
- Control points $P_i \in \mathbb{R}^3$

Bézier surfaces

A Bézier surface is a parametric surface.
The used basis function is the tensor product of Bernstein polynomials :

$$B_{i,n}(t) = \binom{n}{i} * t^i * (1 - t)^{n-i}$$

Expression of a Bézier surface of degree n, m

$$Q(s,t) = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{i,j} B_{i,n}(s) B_{j,m}(t)$$

B-Splines surfaces - 1/2

Data

- Let k & l be two integers (degree of the surface).
- Let m & n be two integers (order of the surface).
- Let $S = \{s_0, \ldots, s_{m+k-1}\}$, $T = \{t_0, \ldots, t_{n+l-1}\}$ two knot vectors, with $s_0 \leq s_1 \leq \ldots \leq s_{m+k-1}$, $t_0 \leq t_1 \leq \ldots \leq t_{n+l-1}$.

- A B-Spline surface is a parametric surface.
  The used basis function is the tensor product of Cox de Boor functions defined by:

$$N_{i,r}(t) = \frac{t - t_i}{t_{i+r-1} - t_i} N_{i,r-1}(t) + \frac{t_{i+r} - t}{t_{i+r} - t_{i+1}} N_{i+1,r-1}(t)$$

B-Splines surfaces - 2/2

Expression of a B-Spline surface of order n,m, degree k,l

$$
\begin{array}{rccl}
Q: & [s_{k-1}, s_m] \times [t_{l-1}, t_n] & \longrightarrow & \mathbb{R}^3 \\
 & (s,t) & \longmapsto & Q(s,t) \\
 & & & = \sum_{i=0}^{m} \sum_{j=0}^{n} P_{i,j} N_{i,k}(s) N_{j,l}(t)
\end{array}
$$

Remark

Fonctions $N_{i,r}(t)$ are piecewise polynomials with compact support.
B-Spline surfaces offer local control of the surface.

Linear program

Definition

- a linear program is an optimization problem :

Minimize (linear cost fonction)
linear constraints

Linear program

Definition

- a linear program is an optimization problem :

$$\begin{gathered} \textit{Minimize} \text{ (linear cost fonction)} \\ \text{linear constraints} \end{gathered}$$

- Example

$$\begin{gathered} \underset{x,y}{Min}\,(5x + 7y) \\ 3x - 9y \leq 2 \\ 8x + 12y \leq 7 \\ -5x + 8y \leq 3 \end{gathered}$$

Our problem of reconstruction

parameterized point cloud

LP fitting ↓ least squares fitting

parametric surface

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Input

- points M_k, $1 \leq k \leq N$
- parameters values s_k, t_k associated to M_k
- basis of functions f_i $1 \leq i \leq n$

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What is an ideal solution ?

$$\forall k, \quad Q(s_k, t_k) = M_k$$

The exact reconstruction system

$$Q(s_k, t_k) = M_k \quad \forall k$$

$$\Longleftrightarrow$$

$$\sum_{i=1}^{n} P_i f_i(s_k, t_k) = M_k \quad \forall k$$

$$\Longleftrightarrow$$

$$\sum_{i=1}^{n} P_i^x f_i(s_k, t_k) = x_k \quad \forall k$$
$$\sum_{i=1}^{n} P_i^y f_i(s_k, t_k) = y_k \quad \forall k$$
$$\sum_{i=1}^{n} P_i^z f_i(s_k, t_k) = z_k \quad \forall k$$

$$\Longleftrightarrow$$

$$A * P^x = X$$
$$A * P^y = Y$$
$$A * P^z = Z$$

Solving the three systems gives an exact reconstruction

x-coordinate system

Let us consider the first system : $A * P^x = X$

In general, there is no exact solution.

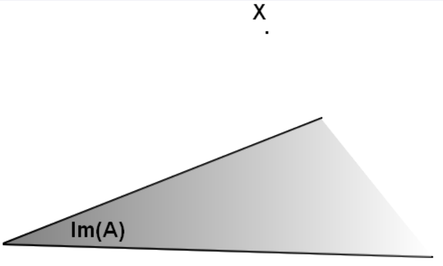

Figure: No solution

Error E^x infered by P^x

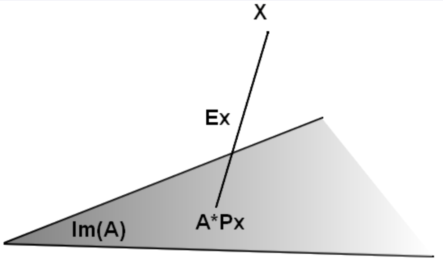

Figure: Error E^x infered by x-coordinates of control points P^x

Least square fitting [2]

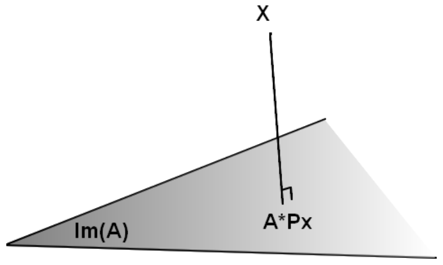

Figure: Least square fitting

Minimize euclidian norm E^x
Orthogonal projection of X onto $Im(A)$

Features of least square fitting

- A distant point can be considered as noise

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- Example : bone excrescence

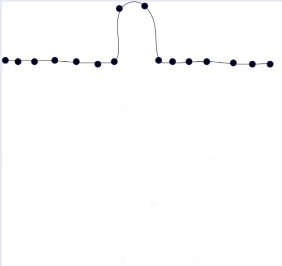

Figure: Bone Excrescence

Features of least square fitting

- A distant point can be considered as noise
- Example : bone excrescence

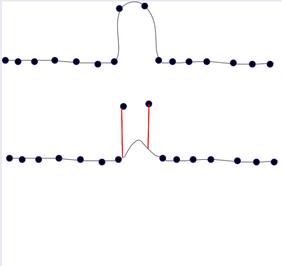

Figure: Bone Excrescence, Least squares fitting

- Least squares fitting minimizes mean error.

Features of least square fitting

- A distant point can be considered as noise
- Example : bone excrescence

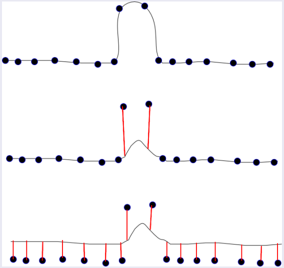

Figure: Bone Excrescence, a better solution according to uniform error

- If we consider uniform norm, there exists better solutions

Our approach : uniform approach

uniform approach

Instead of minimizing euclidian norm of $E^x = A * P^x - X$, we minimize its uniform norm :

$$\underset{P^x}{Min}\left(||E^x||_\infty\right)$$

Toward linear program

$$\left\{ \underset{P^x}{Min}\,(||E^x||_\infty)\right.$$

$$\iff \left\{ \underset{P^x}{Min}\left(\underset{i}{Max}|E_i^x|\right)\right.$$

$$\iff \left\{ \begin{array}{l} \underset{P^x,h}{Min}(h) \\ -h \leq E_i^x \leq +h \;\; \forall i \end{array}\right.$$

$$\iff \left\{ \begin{array}{l} \underset{P^x,h}{Min}(h) \\ -h * \mathbb{1} \leq A * P^x - X \leq h * \mathbb{1} \end{array}\right.$$

Thus, the problem is formulated by a linear program.

Tests

Surface generation

- Bézier surfaces
- B-Spline surfaces
- sphere

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Perturbation

Gaussian noise on

- points M_k
- parameters s_k, t_k

Compare the results

Points M_k

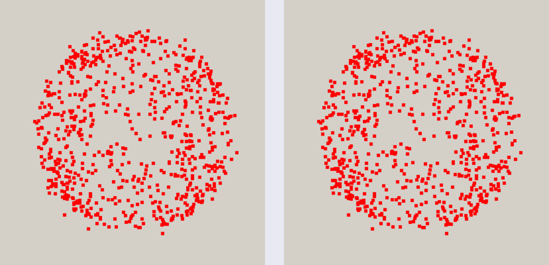

Figure: Results of reconstruction, left LP, right LSF

Compare the results

Points M_k, Surface Q

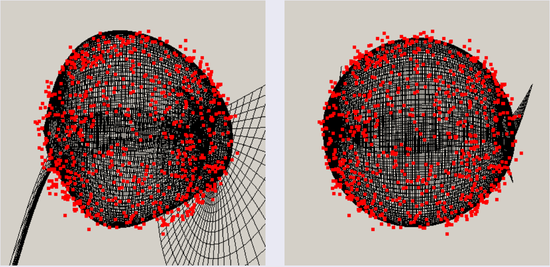

Figure: Results of reconstruction, left LP, right LSF

Compare the results

Points M_k, Surface Q and deviation vectors

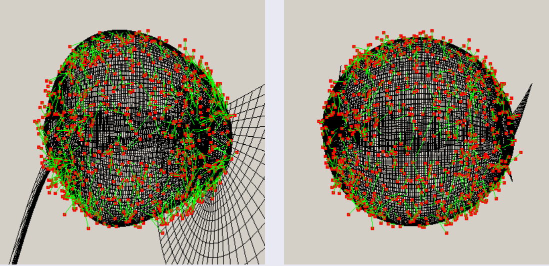

Figure: Results of reconstruction, left LP, right LSF

Compare the results

Data processing

- We pick deviation vectors $\overrightarrow{M_k Q(s_k, t_k)}$
- We compute their euclidian norm
- We put them in an histogram

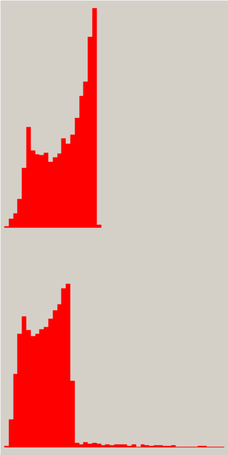

Figure: Favourable case of reconstruction (top LP, bottom LSF)

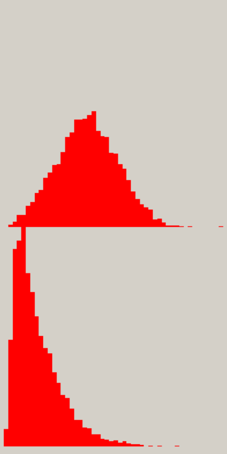

Figure: Unfavourable case of reconstruction (top LP, bottom LSF)

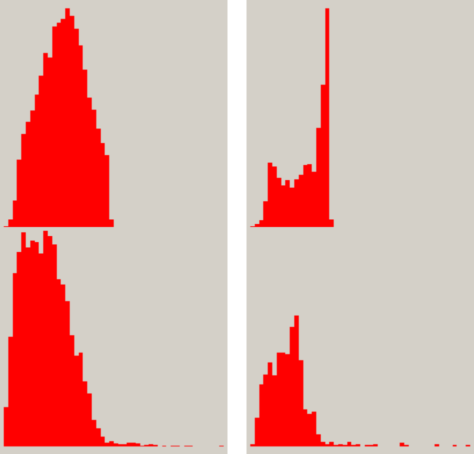

Figure: Usual case of reconstruction (top LP, bottom LSF)

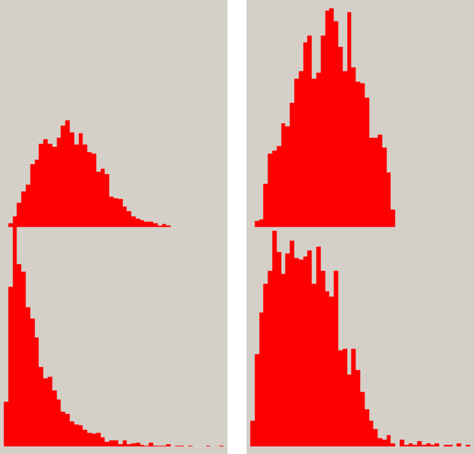

Figure: Usual case of reconstruction with disturbed data (top LP, bottom LSF)

Conclusion

- LP fitting is an alternative to Least square fitting
- Useful for surface reconstruction with a fixed tolerance on the error [3, Weiss & al]

references

[1] Eck, M. and Hoppe, H. (1996). Automatic reconstruction of B-Spline surfaces of arbitrary topological type.
[2] Farin, G. (2002). Curves and surfaces for CAGD: a practical guide.
[3] Weiss, V., Andor, L., Renner, G., and Varady, T. (2002). Advanced surface fitting techniques.

Any questions ?